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Functional equations characterizing differential operators

Results of joint work with Włodzimierz Fechner and Eszter Gselmann.
If $k \geq 2$ is a positive integer, $\Omega \subset \mathbb{R}^N$ is a domain then by the well-known properties of the Laplacian and the gradient, we have

$$\Delta(f \cdot g) = g\Delta f + f\Delta g + 2\langle \nabla f, \nabla g \rangle$$

for all $f, g \in \mathcal{C}^k(\Omega, \mathbb{R})$. Due to the results of König–Milman [1], the converse is also true under some assumptions. Thus the main aim is this talk is to study the equation

$$T(f \cdot g) = fT(g) + T(f)g + 2B(A(f), A(g)) \qquad (f, g \in P),$$

where Q and R are commutative rings and P is a subring of Q, further $T\colon P \to Q$ and $A\colon P \to R$ are additive mappings, while $B\colon R \times R \to Q$ is a symmetric and bi-additive mapping.

References

[1] König, Herman and Milman, Vitali. *Operator Relations Characterizing Derivatives.* Springer, 2018.