Airy spaces and the Baireness of some function spaces

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We shall discuss the following problem: *characterize metrizable separable spaces X for which the space $B_1(X) \subset \mathbb{R}^X$ of functions of the first Baire class on X is Baire*. It turns out that the class of such spaces X is intermediate between the class of λ-spaces and the class of universally meager spaces.

Let us recall that a topological space X is

- a *λ-space* if every countable subset is of type G_δ in X;
- *universally meager* if for every map $f : B \to X$ from a second countable Baire space B the image $f(U)$ of some nonempty open set $U \subset B$ is a singleton;
- *almost analytic* if every countable subset of X is contained in an analytic G_δ-subspace of X;
- *airy* if there exists a countable family $\mathcal{P}$ of infinite subsets of X such that or any $\mathcal{P}$-dense G_δ-sets $A, B \subset X$ the intersection $A \cap B$ is not empty.

A subset $D \subset X$ is *$\mathcal{P}$-dense* if $\forall P \in \mathcal{P}\ \ (P \cap D \neq \emptyset)$.

Theorem. *For a metrizable separable space X consider the conditions:*

(1) *X is a λ-space;*
(2) *the function space $B_1(X)$ is Baire;*
(3) *X is not airy;*
(4) *X is universally meager.*

Then (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4). *If the space X is almost analytic, then* (4) $\Rightarrow$ (1) *and hence the conditions* (1)–(4) *are equivalent.*

This is a joint work with Saak Gabriyelyan.